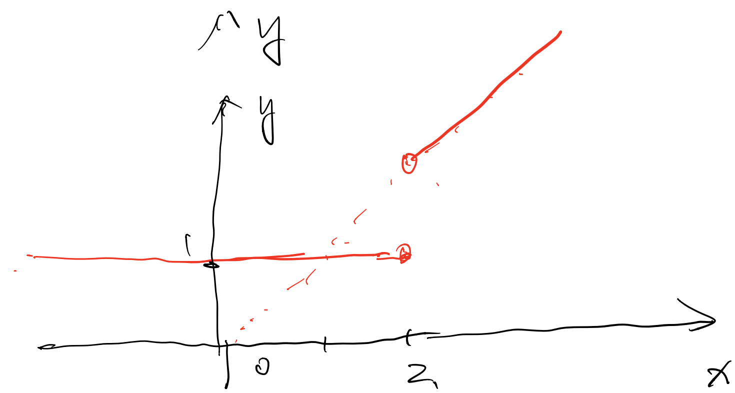
Jan 14

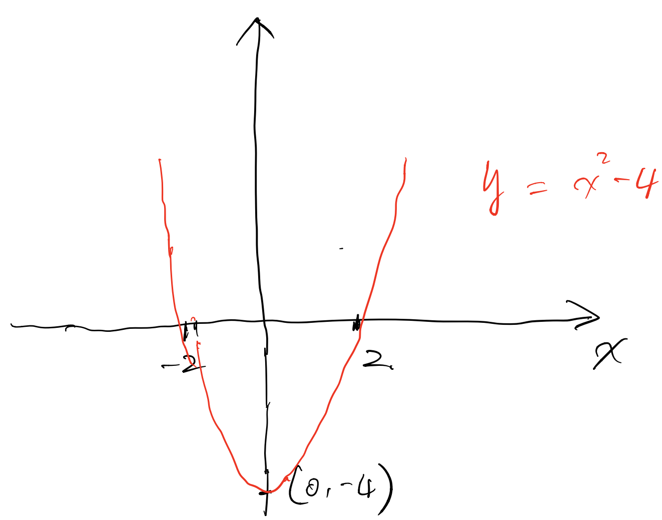
Examples of graphing functions

E.5. "Piecewise linear functions"



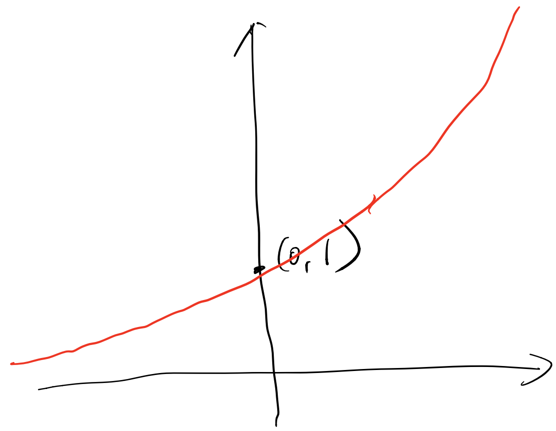
$$f(x) = \begin{cases} 1 & \text{when } x \leq 2 \\ x & \text{when } x > 2 \end{cases}$$

E.9. $f(x) = x^2 - 4$.



E.g.,

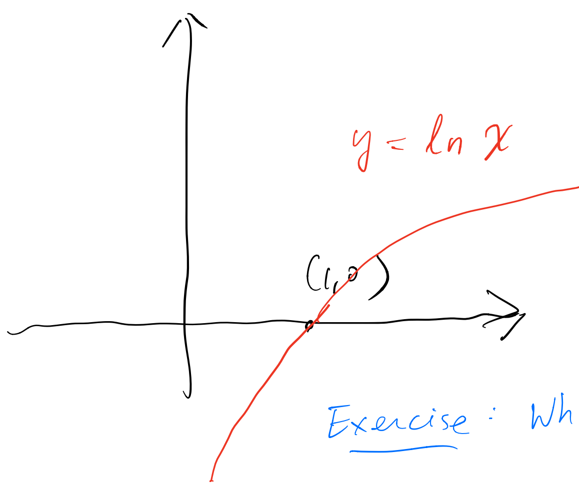
$f(x) = e^x$



E.g., $f(x) = \ln x$

ln : natural log. (base e)

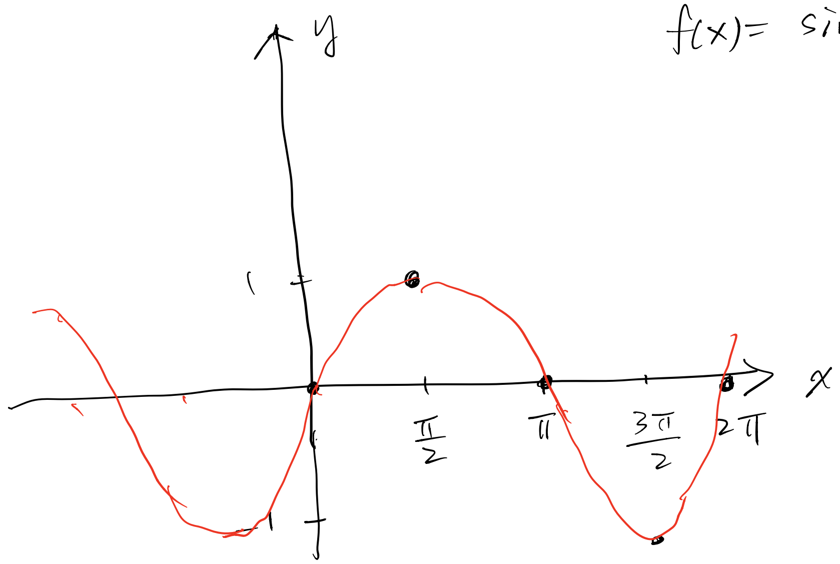
log = ln in math literature and in this course



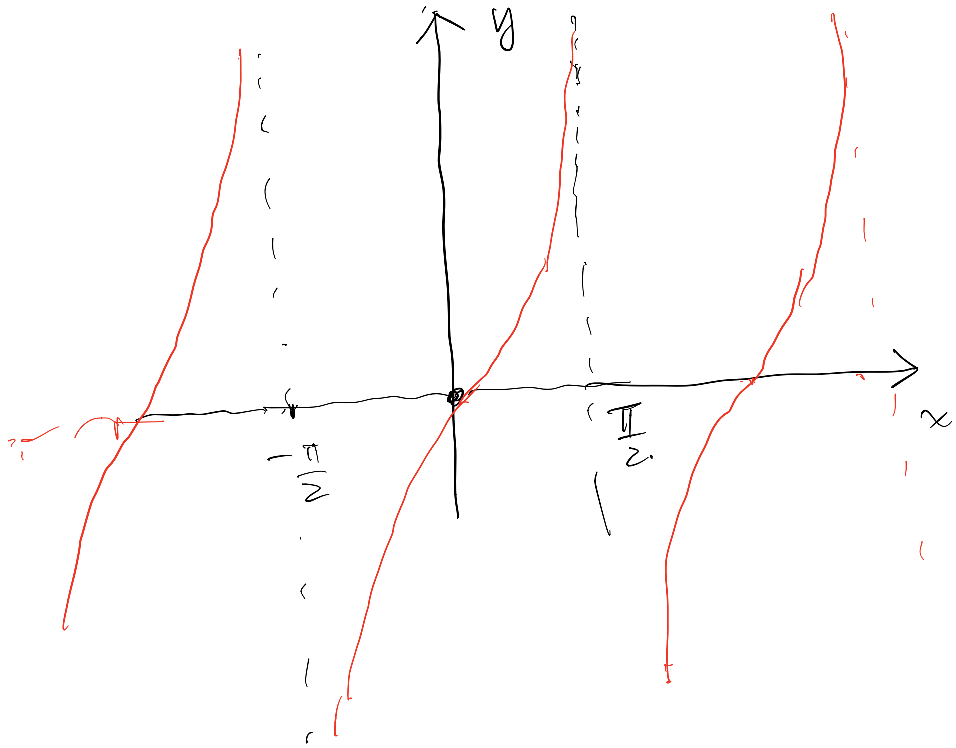
Exercise: What is the natural domain of $\ln x$?

E.g.,

$f(x) = \sin x$

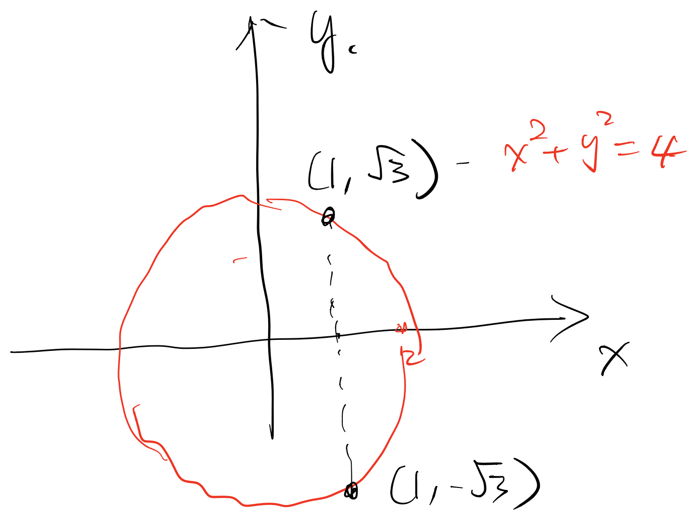


E.g. $f(x) = \tan x = \dfrac{\sin x}{\cos x}$



- Not every curve on the x-y plane is the graph of a function.

E.g.



this circle is not the graph of any function, because there are two points on this circle with the same x-coordinate

1.4.1 Vertical Line Test for Graph

A way to visualize a function is its graph. If f is a real-valued function of one variable, its graph consists of the points in the Cartesian plane $\mathbb{R}^2$ whose coordinates are the input-output pairs for f. In set notation, the graph is

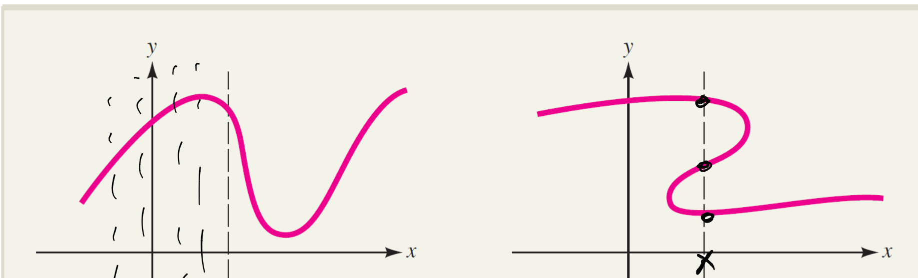
$$\Gamma(f) = \{(x, y) \in \mathbb{R}^2 : x \in \mathbb{R}, y = f(x)\}.$$

Review: Graphing a real-valued function of one variable: [HBSP] 1.2.

Example 1.4.4. linear functions; piecewise linear functions; quadractic functions, exponential and log functions, trig functions.

It is important to realize that not every curve is the graph of a function. For instance, suppose the circle $x^2 + y^2 = 5$ were the graph of some function $y = f(x)$. Then, since the points $(1, 2)$ and $(1, -2)$ both lie on the circle, we would have $f(1) = 2$ and $f(1) = -2$, contrary to the requirement that a function assigns one and only one value to each number in its domain. Geometrically, this happens because the vertical line $x = 1$ intersects the graph of the circle more than once. The vertical line test is a geometric rule for determining whether a curve is the graph of a function.

The Vertical Line Test A curve is the graph of a function if and only if no vertical line intersects the curve more than once:



(a) The graph of a function (b) Not the graph of a function

1.4.2 Some Special Functions

Definition 1.4.2. A piecewise function is defined by more than one formula, with each individual formula defined on a subset of the domain.

Example 1.4.5. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

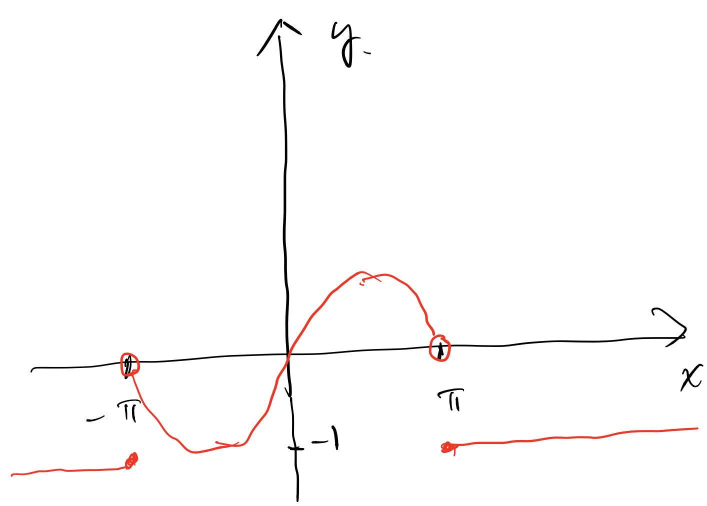
$$f(x) = \begin{cases} 1, & \text{if } x < 0 \\ 2x, & \text{if } x \geq 0. \end{cases}$$

Then $f(-1) = 1$, $f(0) = 0$ and $f(1) = 2$.

piecewise linear function

E.g. Let $f(x) : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} \sin x & \text{when } |x| < \pi \\ -1 & \text{when } |x| \geq \pi. \end{cases}$$



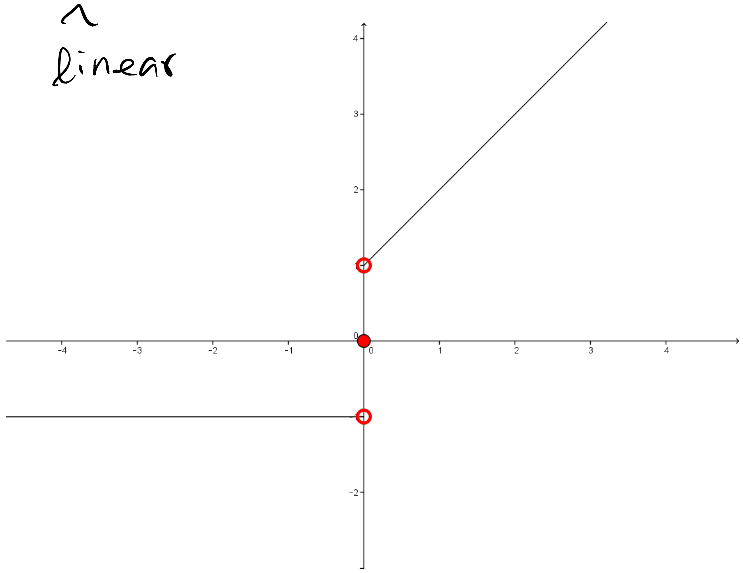
piecewise function, but not piecewise linear

Example 1.4.6. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x+1, & \text{if } x > 0, \\ 0, & \text{if } x = 0, \\ -1, & \text{if } x < 0. \end{cases}$$

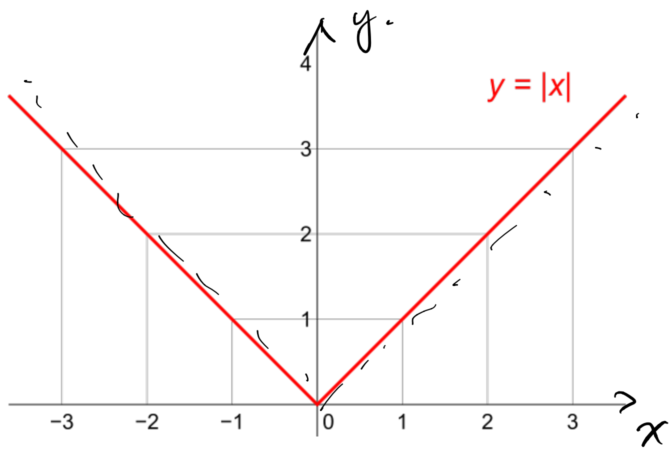
Then f is a piecewise function.

linear



Example 1.4.7. The absolute value function

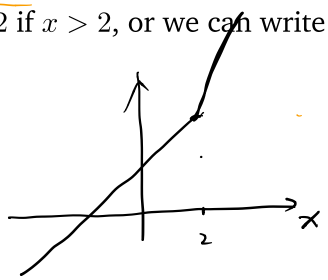
$$f(x) = |x| := \begin{cases} x, & \text{if } x \geq 0, \\ -x, & \text{if } x < 0. \end{cases}$$



actually piecewise linear

Example 1.4.8. Write $f(x) = 2x + |2 - x|$ as a piecewise function.

Solution. Note that $|2 - x| = 2 - x$ when $2 - x \geq 0$, that is $x \leq 2$; and $|2 - x| = x - 2$ when $2 - x < 0$, that is, $x > 2$. Hence $f(x) = 2x + 2 - x = x + 2$ if $x \leq 2$, and $f(x) = 2x + x - 2 = 3x - 2$ if $x > 2$, or we can write



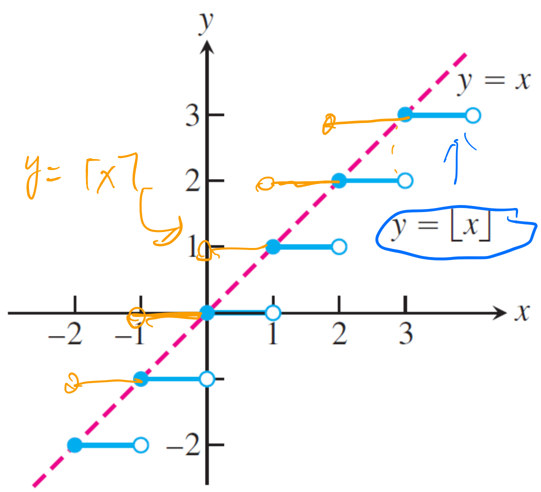
$$f(x) = \begin{cases} x + 2 & \text{if } x \leq 2 \\ 3x - 2 & \text{if } x > 2 \end{cases}$$

$2x + 2 - x$

$2x + x - 2$

■

Example 1.4.9. Define the *floor function* as $\lfloor x \rfloor$ = the largest integer $\leq x$. Then $f(x) = \lfloor x \rfloor$ is a piecewise function.



Exercise 1.4.1. Define the *ceiling function* as $\lceil x \rceil$ = the smallest integer $\geq x$. Sketch the graph of $\lceil x \rceil$.

$\lceil x \rceil$

Exercise 1.4.2. Sketch the graph of

$$f(x) = \begin{cases} x - 2, & \text{if } x > 1, \\ -1, & \text{if } 0 \leq x \leq 1, \\ x^2, & \text{if } x < 0. \end{cases}$$

piecewise function
not piecewise linear.